Suitability of Log-logistic Distribution to Model Water Demand Data

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INTRODUCTION

- A major unresolved problem in the water consumption modelling is identification of a statistical distribution, which best represents water consumption patterns.
- The aim of this research is to assess standard probability distribution in water demand modelling.
- Modelling real water consumption data permit forecasting the probability of occurrences in any demand value & assist planners for future projections.

LITERATURE REVIEW

- Few studies can be found in which, the random variations of demands have been considered.
- Goulter and Bouchart, Xu and Goulter (1990) made an assumption that the demands have a normal distribution.
- Mays (1994) used randomly generated water consumption data using a range of distributions to study the sensitivity of the system's performance to changes in water consumption patterns.
- Khomsi et al (1996). Illustrated that water demand patterns a normal distribution.
- Bowen et al (1993) for residential water demand use patterns in USA results, revealed that the demand data was not distributed normally.
- Surendran and Tanyimboh (2004) addressed the issue and concluded; water demand data fits well in to log logistic distribution when compared to a normal distribution.

Methodology

- Water demand data from Three (3) zones were obtained from a Canadian Water Company for this study.
- Daily demand data were obtained for the last five (5) years (2010-2015)
- In this research, graphical methods were selected for the analysis along with the ***Maximum Likelihood Method*** to generate probability plots using MINITAB statistical package.
- The dataset was analysed using a 95 % confidence interval at a 5% significance level.The goodness of parameter used is the *Anderson Darling test*.

Analysing Data

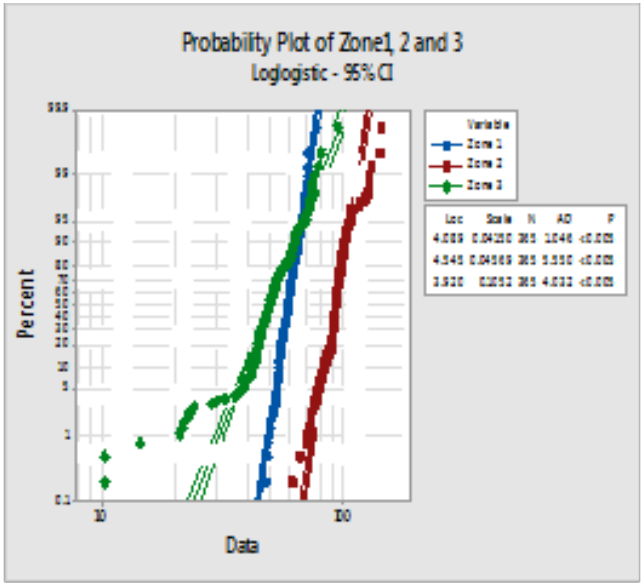
- Data were fitted into four (4) distributions namely **Normal**, **Log nomal**, **Log logistic** & **Weibull**

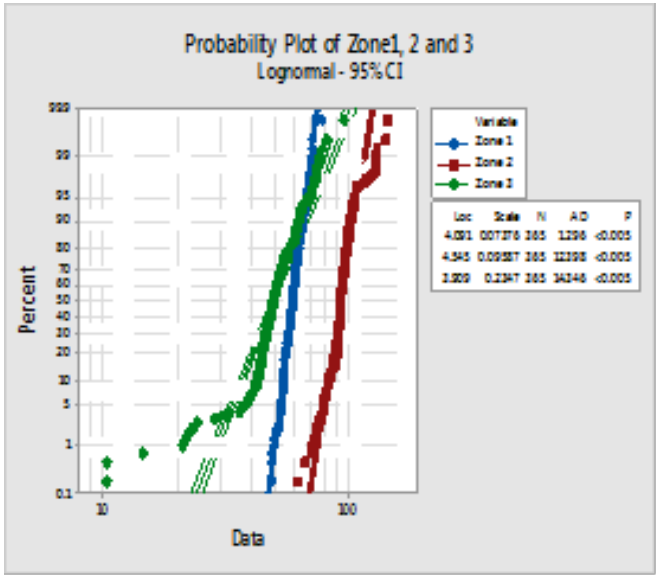
- The *goodness of fit method* was used to see how well data fitted into the particular distribution.

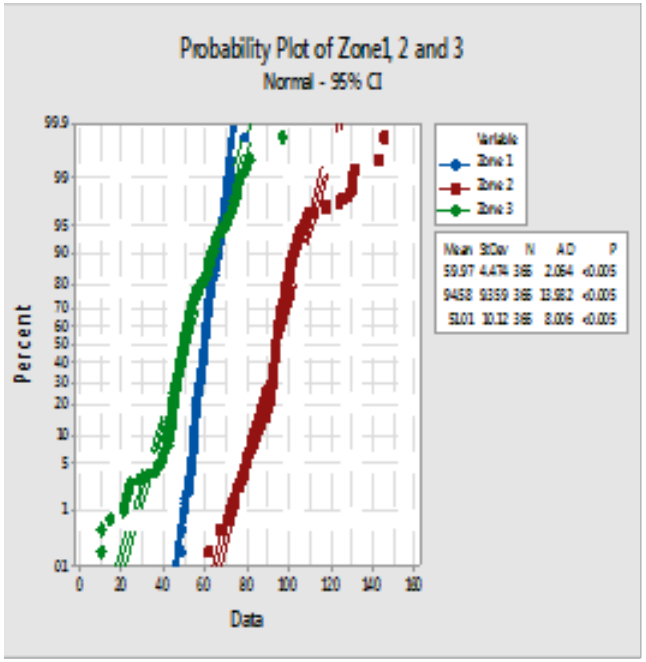
- The appropriateness of the Log logistic distribution for water consumption data was assessed by comparison to the normal and log normal distributions using the Anderson Darling goodness of fit parameter.

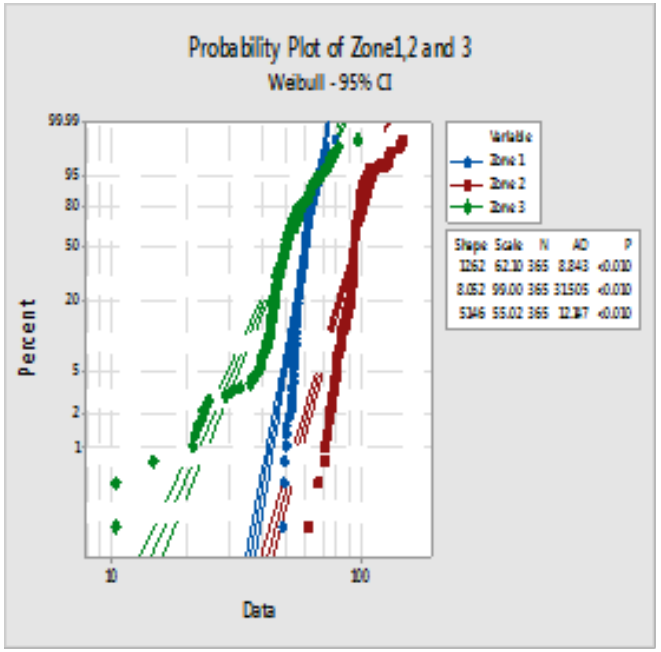
- The *Anderson Darling (AD) statistical method* is a measure of how far the plot points fall from the fitted line in a probability plot. A smaller AD value indicates that the distribution fits the data better.

Results

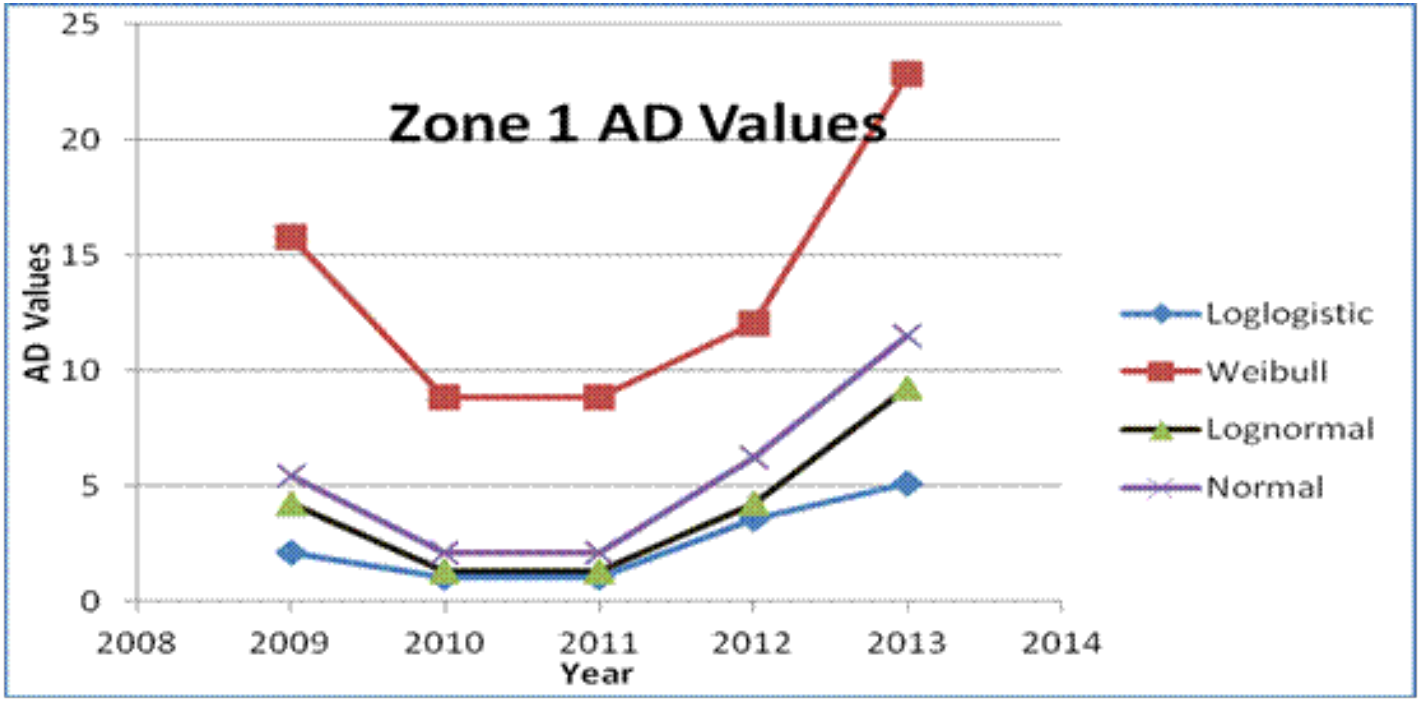

Probability Plot of Zone1, 2 and 3
Loglogistic - 95% CI
Percent
Data
Variable
Zone 1
Zone 2
Zone 3


Probability Plot of Zone1, 2 and 3
Lognormal - 95% CI
Percent
Data
Variable
Zone 1
Zone 2
Zone 3


Probability Plot of Zone1, 2 and 3
Normal - 95% CI
Percent
Data
Variable
Zone 1
Zone 2
Zone 3


Probability Plot of Zone1,2 and 3
Weibull - 95% CI
Percent
Data
Variable
Zone 1
Zone 2
Zone 3

Results


Zone 1 AD Values
AD Values
25
20
15
10
5
0
2008
2009
2010
2011
2012
2013
2014
Year
Loglogistic
Weibull
Lognormal
Normal

Location, scale and shape Parameters

•These parameters will allow the distribution to have a flexibility and effectiveness in modeling applications.

•Shape parameters allow a distribution to take on a variety of shapes depending on the value of the shape parameter.

•The effect of the location parameter is to simply shift the graph to left or right on the horizontal axis.

•The scale parameter describes the stretching capacity of the probability distribution function. If the scale parameter is greater than 1 then it will stretch the probability distribution function.

Parameters to describe the distribution

•Location, shape and scale parameters are also essential to describe the distribution.

Table 1. Shape and Scale parameters for Log logistic distribution

	Zone 1		Zone 2		Zone 3	
	Location	Scale	Location	Scale	Location	Scale
2009 data	4.089	0.0415	4.545	0.0457	3.920	0.105
2010 data	4.089	0.0415	4.545	0.0457	3.920	0.105
2011 data	4.187	0.055	4.446	0.063	4.308	0.142
2012 data	4.098	0.389	4.361	0.041	4.152	0.092
2013 data	4.115	0.0296	4.562	0.054	3.992	0.088

CONCLUSION

- Findings from this study show that distribution patterns for Canada is very similar to UK studies completed in 2002 by Surendran and Tanyimboh.

- The AD values obtained for the Weibull distribution has higher values than other 3 distributions and it is not suitable to model water demand data.

- The study shows that out of four distribution patterns studied, the log-logistic seems to have the lowest AD values and it was the most suitable distribution pattern to standardise when modelling water demand. However, normal and the log-normal distribution also have marginally acceptable AD values.

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Questions?

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